Vector Field Topology in the Context of Separation and Attachment of Flows

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1. Introduction

The interaction of a flow with the objects embedded in it mostly results in attachment or detachment (also separation) of the flow to respectively from the object. Separation and attachment usually appears at isolated points or lines. In steady flows, separation and attachment are related to singularities and separatrices, i.e. vector field topology, of the wall-shear stress field [SGH06]. Consequently, a number of flow visualization techniques employing the concepts of vector field topology for the detection of separation and attachment have been proposed. First, Helman and Hesselink extracted separation and attachment lines (short: feature lines) as separatrices of the wall-shear stress field [HH90]. Later, Kenwright et al. [KHL99] presented a marching method that used the "separatrices" of the linear vector field given in triangle cells with special vector configurations to construct segments of piecewise defined feature lines. Only recently attracting singularities on surfaces have been used for illustrating vortices connected to object boundaries [WTS*07]. These vortices are also connected to separation as they drag the flow away from the surface. Unfortunately, the mentioned methods using separatrices lead to incorrect results if applied to instantaneous vector fields, i.e. snapshots of unsteady flows [SJGH08]. Thus, the discussion of separatrices on surfaces as feature lines applies only to autonomous vector fields throughout the rest of the paper.

In this paper we discuss some important connections between vector field topology on surfaces, the separation of flows and the geometry of the flow embedded object that have not been presented in this way before.

Example The discussion uses a ball immersed in a flow which is shown in figure 1) as central example (simulation by Markus Rütten, DLR Göttingen). The ball has a cylindrical hole through its center and the flow approaches it from one side (lower left in figure 1). The flow's principal direction is parallel to the hole's cylinder axis. Behind the ball, the flow develops two counter-rotating main vortex rings (yellow in fig. 2) and a third vortex very close to the surface which create an alternating pattern of separation and attachment on the surface of the ball. The pattern can be nicely seen in the LIC and topology graph visualization of the wall shear field.

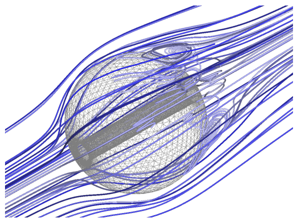

Figure 1: *Streamlines in flow around a ball with hole.*

2. Discussion

Separation and Attachment Lines and Singularities The feature lines are traced out by the separatrices connecting singularities on the surface. Even though the LIC and separatrices both do not have directional cues, the type of the feature line (sep./att.) is deducible from the illustrations in figure 2. The singularities on each line have a generally alternating pattern which consists of either saddles and sinks or saddles and sources. While a saddle does not give any hint on the type of feature line, sinks indicate a separation because flow converging on the surface has to leave it at some point and sources indicate attachment because the diverging flow on the surface attracts fluid from the surrounding volume.

Separation and Attachment Lines and Separatrices Inspecting the images in figure 2 (left and middle) we find separatrices that connect saddles on separation lines with singularities on attachment lines and vice versa. They run mainly orthogonal to the feature lines and parallel to principal flow direction. We have found such separatrices to appear in most datasets we examined. They make the image relatively cluttered and distract the attention from the main separation pattern. Still, they are not wrong with respect to the vector field in each triangle, the detected saddle points imply these separatrices. However, many of them might not be important. This can have two different causes: Firstly, close pairs of singularities with Poincaré index of opposite sign can easily be created and eliminated by small disturbance of the flow. This fact can and has been used for elimination of such singularities (see next section). Secondly, sharp edges, as shown in figure 2 middle upper, can cause singularities in adjacent triangles because the vectors at the vertices of the sharp edge have to be projected into the respective triangles although these triangles have a quite large inclination to each other.

Topology Graph Simplification The clutter observable in figure 2 can be reduced removing unimportant singularities that do not have a large impact on the flow. This has been demonstrated in a number of publications on elimination of close by singularities. It began with the work of Tricoche et al. [TSH01] for vector fields on planar triangulations. Later it has been extended to 3D triangulations and 2-manifolds in general by Chen et al. [CML*07], and Reininghaus and Hotz [RH10] respectively. The methods' usefulness for flow topology in the context of separation and attachment has not been discussed an may vary.

For demonstration purposes (fig. 2 right) we implemented a different but very simple approach. We detected pairs of neighboring triangles that contain singularities with Poincaré indices of opposite sign. Then we take the vectors at the two vertices both triangles have in common and modify them randomly with vectors of increasing norm. During this process we check if the indices of all triangles adjacent to the two vertices. If they all become zero we terminate the search as we have been able eliminate the two singularities with opposite sign in this case. Figure 2 (right) shows an example of such a simplification for a couple of cell pairs. Compared to figure 2 some very close singularities have been eliminated. As one of the two singularities always is a saddle point, we also reduced the number of separatrices and thus the clutter in the image while the feature lines are still visible. Further simplification by elimination of singularities with a certain distance will yield images with even less clutter.

As the singularities close to sharp edges do not result from small disturbances but from the extreme shape of the surface the common simplification approaches are probably not a good choice here. Instead, we suggest to detect sharp edges by surface curvature and exclude all triangles adjacent to sharp edges from the singularity detection or any other feature line detection method. A sharp edge always coincides with separation or attachment (except the flow is perfectly parallel to the edge (unstable)), thus its detection is not needed there.

3. Conclusion

It is known that vector field topology of the wall shear field can help to detect separation and attachment of flows. Although it has not been described in existing literature yet, the common clutter of the topology graph can be reduced by topology simplification and edge detection methods without losing the depiction of the feature line. The type of a feature lines can be determined by the singularity pattern creating the corresponding separatrices.

Finally note that we detected 3D singularities (mostly 3D saddle points) close to any separation and attachment that is not caused by sharp edges of the geometry. This deserves further investigation and is an interesting topic for the discussion at the workshop.

References

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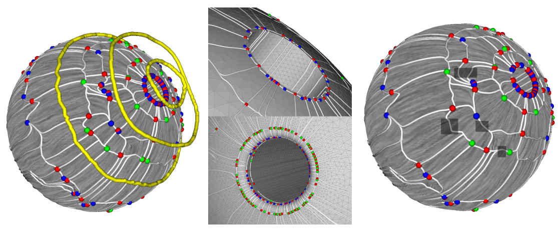

Figure 2: *Separation and attachment in ball example. The flow near the surface of the ball is depicted using LIC, vector field singularities (sinks, sources and saddles) and separatrices (white). The images in the middle show the sharp edge at the outflow end of the hole and the smooth inflow side of the hole. The right image shows the topology of the simplified vector field. The number of singularities and separatrices has been reduced. Some areas where singularities have been eliminated are marked grey.*